Compression Ratio Expansion for Complementary Code Set Compressing a Signal to a Width of Several Sub-pulses

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Abstract— **We present the compression ratio expansion methods for the complementary code set compressing a signal to a width of several sub-pulses. We also proposes the complementary code sets that consist of more that three individual codes and compress a signal to a width of several sub-pulses.**

I. Introduction

The pulse compression is one of the most important techniques for the radar since it could improve range resolution and S/N at the same time. However, there is a problem that unnecessary range sidelobes, which have negative influences on radar's detection performance, are present before and after the compressed pulse. It is therefore the most important issue to reduce range sidelobes. Adopting the complementary code set is one of the most effective methods to eliminate sidelobes when the Doppler effect is small; it, therefore, is used for weather radars and so on. The complementary code set consists of a pair of individual codes, which have the property that the sum of their 2 auto-correlation functions has no sidelobe. Nowadays, There are many attempts to apply it to the communication technology [1][2].

In the field of the pulse compression technique, the coded pulse compression method has excellent merits that using different codes each other can reduce mutual interference between neighbor radars. The more usable pulse compression codes, we have, the more effective, such advantage of code modulation can be [3][4].

On the other hand, the compression ratio expansion methods have been introduced for the complementary code set. Furthermore, the complementary code sets composed of more than 3 individual codes have been proposed. As a result, it has become possible to use the complementary code set at relatively many compression ratios and the freedom degree of the radar signal design becomes large[5][6] .

On the other hand, in the field of the ordinary coded pulse compression using one code, a new pulse compression code to compress a pulse to a width of several sub-pulses was proposed and it was shown that it has some merits compared to the conventional pulse compression code [7]. And then, a new complementary code set to compress a pulse to a width of several sub-pulses, named Widely Pulse-compressing Complementary Code Set (WPCCS), has also been introduced and it has been reported that extremely more WPCCSs exist than the conventional complementary code set in the region of relatively small compression ratio. Furthermore, other several interesting properties about WPCCS has been reported. First of all, there exist much more complete WPCCSs compared to the complete conventional complementary code set. A pair of complementary code set is defined to be complete if the sum of cross-correlation functions between the mate codes of each complementary code set is always zero for any time shift. It is also noted that, in the WPCCS, there are long complex continuous networks connected by the relationship between 2 complementary code set that they construct a complete complementary code set jointly, while there are only isolated complete complementary codes in the conventional. This fact means that, using such complete WPCCSs on several adjoining radars and synchronizing the pulse transmission, it is possible to suppress the mutual interference even on the same frequency. Namely, the effective use of the frequency resource is possible. It has been also reported that the auto-correlation sidelobe of an individual code for the WPCCS is smaller than that for the conventional. Using such WPCCSs, it is possible to reduce the sidelobe produced by the Doppler frequency. Consequently, it has been clarified that the WPCCS has several interesting properties compared to the conventional complementary code set. However, there is only one method that extends the compression ratio of the WPCCS. So, the merits of the WPCCS is not sufficiently displayed in the region of large compression ratio [8].

In this paper, we show that the compression ratio expansion method for the conventional complementary code set, which was introduced in the article [5], is also applicable for the WPCCS. Further, we introduce that the WPCCS composed of more than 3 individual codes can be constructed by the same method for the conventional complementary code set shown in the article [6]. It also gives the compression ratio expansion method of the WPCCS simultaneously. As a result, in the large compression ratio region, the number of the WPCCS becomes

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14. ABSTRACT

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extremely larger and the freedom degree
design becomes much larger, which make
the important merits of the WPCCS modu

II. DEFINITION

The following briefly introduces the
tations used in this paper. The cross c
individual code A and B is denoted by
is a time shift. The conventional compl
is defined by Eq.(1). On the other han
defined by Eq.(2) [8]. Fig.1 shows the
shape of the WPCCS. The parameter l ir
called "compression parameter" from now
wideness of the compressed pulse bottom
ratio approximately corresponds to n/l in
while it is exactly n in case of the conventic
code set, where n is code length. The com
proposed at first by M.J.E.Golay, which i
individual codes, corresponds to the case v
in Eq. (1). After that, the complementary
of more than 3 individual codes was prop
hand, in the WPCCS of Eq. (2), only one c
to 2 in Eq. (2) was investigated before. T
more than 3 in Eq. (2) is proposed in the

$$\sum_{i=1}^{p} \psi_{A_iA_i}(k) = \begin{cases} 0, & k \\ pn, & k \end{cases}$$

$$\sum_{i=1}^{p} \psi_{A_iA_i}(k) = \begin{cases} 0, & l \le |k| \\ nonzero & 0 < |k| < l \\ pn, & k = 0 \end{cases} \quad (2)$$

III. PROPERTY OF WPCCS

In this section, the several interesting properties of the WPCCS reported before is introduced. First of all, it should be noted that extremely more WPCCSs exist than the conventional complementary code sets in the region of relatively small compression ratio. Furthermore, there exist much more complete WPCCSs compared to the complete complementary code set of the conventional. A pair of complementary code set is defined to be complete if the sum of cross-correlation functions between the mate codes of each complementary code set is always zero for any time shift, i.e. a pair of complementary code set (or a pair of WPCCS), $\{A_i; 1 \le i \le p\}$ and $\{B_i; 1 \le i \le p\}$ is defined to be complete when it satisfies Eq. (3). It is also an interesting characteristics that, in the WPCCS, there are long complex continuous networks connected by the relationship between 2 complementary code set that they construct a complete complementary code set jointly as shown in Fig.2, while there are only isolated complete complementary codes in the conventional. This fact means that, using such WPCCSs on several adjoining radars and synchronizing the pulse transmission, it is possible to suppress the mutual interference even on the same frequency. Namely, the effective use of the frequency resource is possible. It has been also shown that the auto-correlation sidelobe of

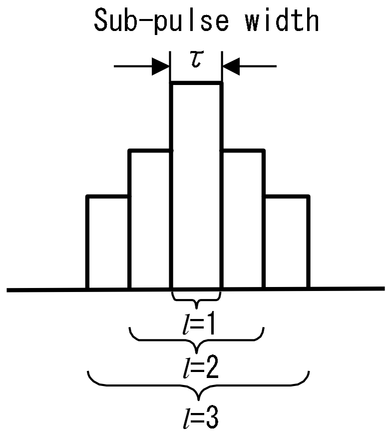


Fig. 1. Compressed Pulse Shape

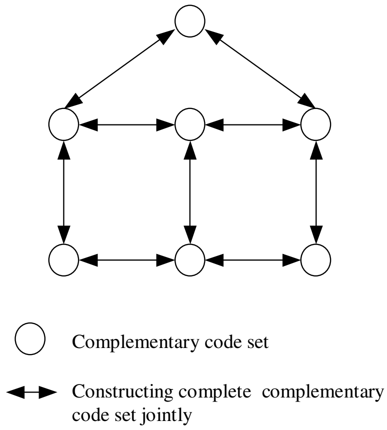


Fig. 2. Compressed Pulse Shape

an individual code for the WPCCS is smaller than that for the conventional. Then, it is possible to reduce the sidelobe produced by the Doppler frequency [8].

Consequently, it has been clarified that the WPCCS has several advantages compared to the conventional complementary code set. However, there is only one method that extends the compression ratio of the WPCCS. So, the merits of the WPCCS is not sufficiently displayed in the region of large compression ratio.

$$\sum_{i=1}^{p} \psi_{A_iB_i}(k) = 0 \quad (3)$$

IV. COMPRESSION RATIO EXPANSION METHOD

In this section, it is shown that the compression ratio expansion method for the conventional complementary code set proposed in the article [5] is applicable also for the WPCCS. Firstly, let's define some notations. A^B represents the code with length nm defined by Eq.(4) when the code A with length n and code B with length m are given.

$$C = A^B = ((b_1A)(b_2A)\cdots(b_mA)) \quad (4)$$

Theorem 1 When the binary code set $\{A_1, A_2\}$ is the complementary code set, i.e., satisfies Eq. (1) and its length is n, further the code set $\{B_1, B_2\}$ is the WPCCS, i.e., satisfies Eq. (2) and its length and compression parameter are respectively m and l. Then, the code set $\{C_1, C_2\}$ given by Eq.(5) is the WPCCS whose length and compression parameter is respectively $2nm$ and l. Where, $\widetilde{A}$ is the code obtained by arranging the code elements of A inversely.

$$\{C_1, C_2\} = \{B_1^{A_1} B_2^{\widetilde{A}_2}, B_1^{A_2} B_2^{-\widetilde{A}_1}\} \tag{5}$$

`Proof`

Let define k and r integers and let s defined as $s = k \mod m$, i.e. $k = rm + s$, then,

$$\begin{aligned}
&\psi_{C_1C_1}(k) + \psi_{C_2C_2}(k) \\
&= \psi_{B_1B_1}(s)\{\psi_{A_1A_1}(r) + \psi_{A_2A_2}(r)\} \\
&+\psi_{B_2B_2}(s)\{\psi_{\widetilde{A}_2\widetilde{A}_2}(r) + \psi_{\widetilde{A}_1\widetilde{A}_1}(r)\} \\
&+\psi_{B_1B_1}(-m+s)\{\psi_{A_1A_1}(r+1) + \psi_{A_2A_2}(r+1)\} \\
&+\psi_{B_2B_2}(-m+s)\{\psi_{\widetilde{A}_2\widetilde{A}_2}(r+1) + \psi_{\widetilde{A}_1\widetilde{A}_1}(r+1)\} \\
&+\psi_{B_2B_1}(s)\ \{\psi_{\widetilde{A}_2A_1}(-n+r) + \psi_{(-\widetilde{A}_1)A_2}(-n+r)\} \\
&+\psi_{B_2B_1}(-m+s) \\
&\{\psi_{\widetilde{A}_2A_1}(-n+r+1) + \psi_{(-\widetilde{A}_1)A_2}(-n+r+1)\} \\
&= \psi_{B_1B_1}(s)\{\psi_{A_1A_1}(r) + \psi_{A_2A_2}(r)\} \\
&+\psi_{B_2B_2}(s)\{\psi_{\widetilde{A}_2\widetilde{A}_2}(r) + \psi_{\widetilde{A}_1\widetilde{A}_1}(r)\} \\
&+\psi_{B_2B_1}(s)\ \{\psi_{\widetilde{A}_2A_1}(-n+r) - \psi_{\widetilde{A}_1A_2}(-n+r)\} \\
&+\psi_{B_2B_1}(-m+s) \\
&\{\psi_{\widetilde{A}_2A_1}(-n+r+1) - \psi_{\widetilde{A}_1A_2}(-n+r+1)\}
\end{aligned}$$

$$\begin{aligned}
&= \psi_{B_1B_1}(s)\{\psi_{A_1A_1}(r) + \psi_{A_2A_2}(r)\} \\
&+\psi_{B_2B_2}(s)\{\psi_{\widetilde{A}_2\widetilde{A}_2}(r) + \psi_{\widetilde{A}_1\widetilde{A}_1}(r)\} \\
&= \begin{cases} 2n\{\psi_{B_1B_1}(s) + \psi_{B_2B_2}(s)\} & |k| < m \\ 0 & |k| \geq m \end{cases}
\end{aligned} \tag{6}$$

Eq.(6) indicates that the code set $\{C_1, C_2\}$ also satisfies Eq. (2) and its compression parameter is l if the code set $\{B_1, B_2\}$ satisfies Eq. (2) and its compression parameter is l.

`Proof End`

The concrete example is shown below.

Let $A_1 = (0, 0), A_2 = (0, 1)$ and $B_1 = (0, 1/2, 0), B_2 = (0, 1/2, 1)$,

Then $\{A_1, A_2\}$ satisfies Eq. (1) and $\{B_1, B_2\}$ satisfies Eq. (2). $\{C_1, C_2\}$ is obtained from $\{A_1, A_2\}$ and $\{B_1, B_2\}$ by Eq.(5) as follows.

C_1=(0, 1/2, 0, 0, 1/2, 0, 1, 3/2, 0, 0, 1/2, 1)

C_2=(0, 1/2, 0, 1, 3/2, 1, 1, 3/2, 0, 1, 3/2, 0)

Then, (C_1, C_2) satisfies Eq. (2). Fig.3(a) shows the compressed pulse shape of (C_1, C_2).

V. Complementary Code Set Composed of Several Codes

In the WPCCS, only one case when $p = 2$ in Eq. (2) was investigated before. The case when p is more than 3 in Eq. (2) is proposed in this section. Namely, It is shown that the WPCCS composed of more than 3 individual codes can be constructed by the method, which was proposed in the article [2], to construct the conventional complementary code set composed of more than 3 individual codes. It is also shown that the method gives the compression ratio expansion way of the WPCCS simultaneously. First of all, let's define some notations. When the code set $X = \{A_i; 1 \leq i \leq p\}$ and the matrix with q rows and p columns shown in Eq. (7) are given, HX represents the code set whose element codes are each rows of the matrix shown in the right side of Eq.(8).

$$H = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1p} \\ h_{21} & h_{22} & \dots & h_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ h_{q1} & h_{q2} & \dots & h_{qp} \end{pmatrix} \tag{7}$$

Where $|h_{kl}| = 1$

$$\begin{aligned}
Y &= HX \\
&= \begin{pmatrix} (h_{11}A_1)(h_{12}A_2) & \dots & (h_{1p}A_p) \\ (h_{21}A_1)(h_{22}A_2) & \dots & (h_{2p}A_p) \\ & \vdots & \\ (h_{q1}A_1)(h_{q2}A_2) & \dots & (h_{qp}A_p) \end{pmatrix}
\end{aligned} \tag{8}$$

Theorem 2 When $X = \{A_i; 1 \leq i \leq p\}$ is the WPCCS whose compression parameter is l, and all columns of the matrix with q rows and p columns shown in Eq. (7) are orthogonal each other. Then, the code set Y given by Eq.(8) is also the WPCCS whose compression parameter is l.

`Proof`

First of all, Let's investigate 2 matrices shown below.

$$\begin{pmatrix} h_{1r}A_r \\ h_{2r}A_r \\ \vdots \\ h_{qr}A_r \end{pmatrix} and \begin{pmatrix} h_{1s}A_s \\ h_{2s}A_s \\ \vdots \\ h_{qs}A_s \end{pmatrix}$$

The sum of the q cross correlations between two codes which are respectively the same row of the right matrix and left matrix is easily calculated as follows.

$$\begin{aligned}
&\sum_{i=1}^{q} \psi_{(h_{ir}A_r)(h_{is}A_s)}(k) = \sum_{i=1}^{q} h_{ir}h_{is}^{*}\psi_{A_rA_s}(k) \\
&= q \cdot \delta_{rs} \cdot \psi_{A_rA_s}(k)
\end{aligned}$$

Therefore, when each row of Y given by Eq. (8) is represented by B_i. Namely, $B_i = (h_{i1}A_1)(h_{i2}A_2)\dots(h_{ip}A_p)$. Then, Eq.

(9) is obtained.

$$
\begin{aligned}
&\sum_{i=1}^{q} \psi_{B_i B_i}(k) \\
&= q \sum_{i=1}^{q} \psi_{A_r A_r}(k) \qquad (9
\end{aligned}
$$

Proof End

Especially, it is noted that the WPCCS composed of mor than 3 individual codes is obtained from that composed of : individual codes when $p = 2, 3 \leq q$ in Eq. (8). Moreovei the theorem 2 gives the compression ratio expansion metho for the WPCCS. Especially, it gives the compression rati expansion method for the WPCCS composed of 2 individua codes when $p = 2, q = 2$.

The concrete example is shown below. Let A_1 = $(0, 1/2, 0), A_2 = (0, 1/2, 1)$ and

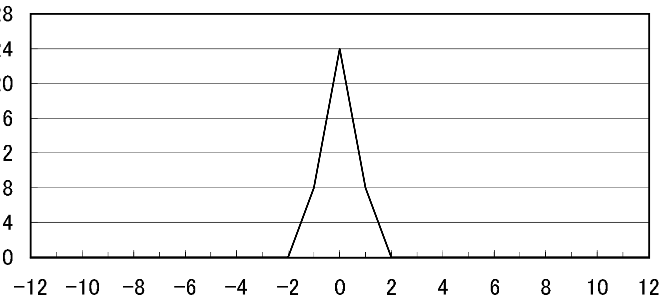
$$
H = \begin{pmatrix} 1 & i \\ 1 & -i \\ -1 & 1 \\ -1 & -1 \end{pmatrix}, Y = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix}
$$

Then, from Eq. (8),
$B1 = (0, 1/2, 0, 1/2, 1, 3/2)$
$B2 = (0, 1/2, 0, 3/2, 0, 1/2)$
$B3 = (1, 3/2, 1, 0, 1/2, 1)$
$B4 = (1, 3/2, 1, 1, 3/2, 0)$
$(B1, B2, B3, B4)$ satisfies Eq. (2). Fig.3(b) shows the compressed pulse shape of $(B1, B2, B3, B4)$.

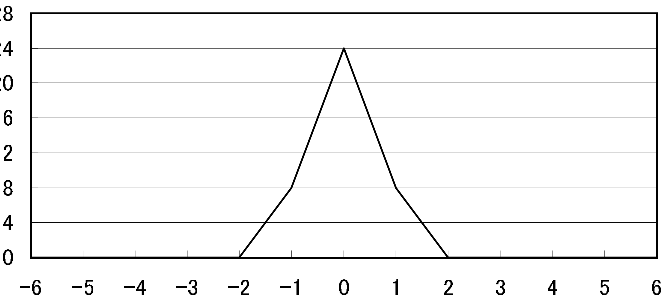
VI. Conclusion

We have proposed the compression ratio expansion method for the WPCCS. Further, the paper introduced that the WPCCS composed of more than 3 individual codes can be constructed. It also gives the compression ratio expansion method of the WPCCS at the same time. There was only one compression ratio expansion method for the WPCCS before and it makes only one WPCCS of larger compression ratio from the one with smaller compression ratio. So the number of the WPCCS didn't increase though the compression ratio increased before. However, we have given several compression ratio expansion methods for the WPCCS. Further, they can construct several WPCCSs with larger compression ratio from the one with smaller compression ratio. As a result, in the large compression ratio region, the number of the WPCCS becomes extremely larger and the freedom degree of the radar signal design becomes much larger, which make it possible to use the important merits of the WPCCS modulation.

The remaining problem is to investigate whether other merits of the WPCCS obtained in the region of small compression ratio can be also obtained in the region of the larger compression ratio.



(a) Expansion result of an example for Proposition 1



(b) Expansion result of an example for Proposition 2

Fig. 3. Compressed Pulse Shape of Example

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